

# The enactive equation: exploring how multiple external representations are integrated, using a fully controllable interface and eye-tracking

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**Abstract**—**Representational competence (RC), defined as “the ability to simultaneously process and integrate multiple external representations (MERs) in a domain”, is a marker of expertise in science and engineering. However, the cognitive mechanisms underlying this ability and how this ability develops in learners, is poorly understood. In this paper, we report a fully manipulable interface, designed to help school students develop RC. Further, as the design emerged from the application of distributed and embodied cognition theory to the RC problem, the design also seeks to shed light on the cognitive mechanisms underlying the integration of MERs. Here we report a preliminary eye and mouse tracking study, which sought to develop a detailed understanding of how students interacted with our interface, under self and text-guided exploration conditions. We also examined how the interaction process related to students' ability to integrate the representations in the interface. Results highlighted several desirable student behaviors, and potential points of modification of the interface to improve integration of MERs.**

**Keywords**- *multiple external representations, representational competence, distributed cognition, embodied cognition, equations, graphs*

## I. INTRODUCTION

Multiple external representations (MERs), such as models of phenomena, equations, and graphs, are used extensively in science and engineering for concept, phenomenon and procedure understanding, problem solving, modeling and design [1][2][3][4][5]. Representational competence (RC) is defined as “the ability to simultaneously process and integrate MERs in that domain” [7]. Several studies report difficulties in student learning, owing to problems in working with MERs (see [7] for a review). Studies comparing experts and novices in various domains show clear differences between the two groups in their understanding, usage and generation of MERs [7][8][9][10][13]. Further, conceptual understanding is tightly correlated with learners' ability to generate multiple representations of concepts, situations and phenomena, and transform between these representations [2].

Students understand and are able to use and generate graphs and equations [11][12] independently. However students often have difficulty understanding how the two representations are related and can be used together [13][14][15][16], and possibly how they relate to the phenomenon. These results indicate that there is a clear need for development of RC among students.

Computer interfaces have been used extensively in science and engineering learning [17][27] typically for the learning goals of improving conceptual, phenomenon and procedural understanding using MERs. Interfaces specifically targeting the development of RC (some examples are [24][25][26]) among learners are few and far apart[7]. The effectiveness of available computer interfaces for learning, particularly learning RC, has been mixed [4][17][27]. One possible reason for this is that interface design in these domains is currently guided by information processing theories of cognition, wherein the role of the interface is to decrease the learner's cognitive load, particularly working memory load[4][23][27]. However emerging theories of cognition, such as distributed and embodied cognition, postulate that the roles played by external representations are wider than decreasing cognitive load [30]. For instance, external representations can support operations that are difficult, and sometimes impossible, to do in imagination [30][31]. Further, actions could be a way of promoting integration [32]. Understanding the RC problem using these new approaches to cognition could help in developing novel interaction designs that help students integrate MERs. Here we report the design of an interface where this approach was followed.

## II. DESIGN PROBLEM AND APPROACH

The generic case of integration of MERs involves a physical system (such as a pendulum or a falling object), or its description, an equation capturing the behaviour of this system, and graphs that display the equation's output for some sets of values. The learner needs to develop an integrated internal representation of the three modes - the phenomenon, its equation and the graphs. An indicator of this integration is the ability to transform smoothly between the three modes. This is difficult, because it requires transformations between spatial and numerical modes (e.g. graph and equation), as well as dynamic and static modes (e.g. phenomenon and equation)[7]. Further, students need to understand how the values in the equation get translated into a graph, which is also a dynamic entity, even though it is displayed as a static picture. Thus, to integrate the MERs, the student needs to "unfreeze" these static representations, by generating their dynamic behavior in imagination, and then connect these dynamics with the dynamic behavior of the phenomenon. Conversely, students also need to be able "freeze" the behavior of real-world systems into equations. Finally, when they start computer programming, students need to understand that the equation can also be a *controller* of

the physical system, meaning that changing the numbers in the equation can change the way the physical system behaves.

In this paper, we present the design of a fully manipulable interface with interconnected MERs, which tries to make transparent: (i) the idea of equation and graph as dynamic entities (ii) the idea of equation as a controller of systems, and (iii) different numerical-spatial and dynamic-static transformations. The novelty of our design is that, unlike simulation models with similar elements (such as Netlogo[37] and PhET[38]), our design is derived from basic research, particularly education research examining RC, and recent cognitive science theories and models, including distributed and embodied cognition. These theories investigate the cognitive roles played by different kinds of representations and their underlying cognitive/neural mechanisms [30][31][32] [29][36].

One feature derived from basic cognition research is the full manipulation of the interface, which seeks to promote integration of MERs. This link is derived from an embodied cognition idea - that actions and manipulation, i.e. motor control, requires integrating multiple cognitive and perceptual inputs, and feedback loops. This suggests that actions and manipulations performed on MERs in an interface would trigger/prime the neural processes involved in integration of inputs; thus it would help in integrating the multiple representations as well. This line of thinking led to making the equation components manipulable. This also introduces the controller role of the equation, a feature not seen in standard simulation models. In this design, students control and 'enact' the equation, and integration is hypothesized to result from this control feature. Thus testing the development of RC based on our design also involves testing this hypothesis, and by extension, the cognitive theory that underlies it. Applying this theory to our interface targeting the physical system of a simple pendulum leads to features such as full learner manipulation of the pendulum via clicking and dragging, controlling the equation parameters using vertical sliders, and complete interconnection between the three modes. By contrast, a PhET pendulum simulation [38] does not have the equation and graph, and there is only one interaction on the pendulum, while the other variable is manipulated via horizontal sliders.

The interface is designed for self-learning by a grade 7 student, and includes specific tasks that encourage exploration, as well as tasks testing RC. After developing a stable initial prototype of the interface, we evaluated its usability and learnability through a two-group controlled study, primarily to address a question that emerged during design: is an interface with guidance better, in terms of exploration and integration, than an interface without guidance? To test this, one group (text-guided) received an interface which had text instructions of how to use the various manipulable features on the interface (eg, sliders). The other group (self-guided) received an interface without these instructions. The research questions (RQs) of this particular study were:

1. What is the difference in student exploration of the interface, particularly in terms of manipulation/control, in the text-guided and self-guided conditions?

2. What is the difference between student exploration of the interface before the tasks are presented and during tasks?

To understand the interaction *process* in detail, we recorded student eye movements and mouse clicks using an eye-tracker, and also captured video as well as unstructured observations by the experimenter.

A first level analysis of the data showed that students in the text-guided condition performed more expert-like exploration than students in the self-guided condition (RQ1) and that student exploration markedly increases after presentation of the task in both text-guided and self-guided conditions (RQ2). The results also pointed to some limitations of our design. Future work includes refining our prototype based on these results and performing a larger controlled study.

### III. RELATED WORK

Student difficulties in science and engineering, stemming from problems in using, linking and transforming between MERs, are well documented in the literature ([7] has a review). These difficulties have been explained by arguing that the conceptual organization of the domains consists of MERs of different types, and at different levels, which, along with the limited capacity of working memory, puts a high cognitive load on the learner [6].

In order to reduce cognitive load and improve students' conceptual understanding by linking and transforming between representations, interventions have focused on the use of computer interfaces [4][17][27]. The functions of MERs in these interfaces include using MERs in complementary roles, MERs to constrain interpretation, and MERs to construct deeper understanding [4]. Such interfaces are designed to support learners transformation across MERs using two types of features: (i) an integrated presentation of MERs of concepts, phenomena and procedures on the same screen, which reduces the split-attention effect [22] and (ii) dynamic linking of representations [20][23] which reduces cognitive load. There have been several studies comparing the learning effects of different ways of presenting information on the interface [17][20][21][22][23][27]. It has been found that the hypothesized learning benefits of such interfaces have not been fully realized [4]. For instance, the learning effects of dynamic linking have not been clearly established [23]. However, supporting our design, it was found that learning improves when actively manipulating the MERs to produce an integrated format, compared to observing an already integrated representation [22].

While the above discussed literature focuses on improving student conceptual understanding via MERs, there is a dearth of research which focuses directly on the development and assessment of RC using computer interfaces [24][25][26]. For instance, a biomolecular visualization software was used in a college biology class to improve students' RC [26]. Pen-and-paper computing was used in an engineering classroom to teach students good representational practices in a participatory learning environment where students learnt to create, share, record and reflect on representations [24] by incorporating technology into the learning environment. By examining the student use of a multi-

representational molecular mechanics animation using eye-tracking and verbal protocol data, researchers show that students mainly use the graph and model representations in the animation and often ignored the equations [25].

The role of tangible interfaces in learning, based on theories of grounded cognition, has been reviewed in [28], which presents an analytical framework for thinking about learning with tangible interfaces. However, the literature offers no consensus on how representations should be combined in tangible interfaces for effective integration, the benefits of various approaches, or the cognitive effects of combining representations [28].

Within cognitive science, the integration of concepts, as well as MERs, is a difficult and poorly understood problem, and has only recently gained some attention even in the philosophy of science [33]. Integration of concepts and MERs requires generating dynamic entities/processes (such as molecular structure or reaction kinetics) in imagination[7], based on external representations, and then changing these imagined entities/processes through changes in the external representation, and vice versa. From a cognition perspective, integration of MERs thus crosses the internal-external divide, and requires the application of recent theoretical frameworks such as distributed cognition, which treat cognition as distributed across people and representations/artifacts. Further, to understand the process of integration of MERs used in science and engineering, both the detailed inner structure of complex and dynamic entities/processes, and the cognition mechanisms that are involved in imagining such complex entities/processes, are needed. Recent research seeks to apply insights from embodied and enactive theories of cognition, particularly common coding and tool use[35], to account for how building and manipulation of external models could lead to conceptual change and discovery[32], and explores how this framework could be used to develop interfaces for graduate-level learning in systems biology [34].

The design research reported here builds on these theoretical and application projects, and seeks to extend these ideas further. We attempt to bridge the gaps in the above reported research, by designing an interface specifically targeting the development of RC (and not conceptual understanding), based on a theoretical framework derived from models of distributed, embodied and enactive cognition. Specifically, we use the idea that action (manipulation) and control are central to integration of representations. Thus our work differs from existing approaches to the design of interfaces (i) in the learning goals we are targeting, (ii) the theoretical framework which guides our design decisions towards facilitating integration and (iii) the possibility of using our design to test embodied/enactive theories of representation. We elaborate on the process of design of our interface in the following sections.

#### IV. DESIGN OF INTERFACE

Firstly, we decided on the concept of oscillation of a simple pendulum as the medium to examine the development of RC. This is because, ignoring the small angle approximation and damping, the pendulum is a physical system with simple dynamics and a

trigonometric equation that is easy to understand. This was appropriate for us as we didn't want the complexity of the concept to interfere with the learners' integration of representations.

The design of the interface evolved through three iterations, and the main principle of our design was manipulability and sense of control. A complete list of design principles from distributed and embodied/enactive cognition theory [30][31][32] and their operationalization is shown in Table 1.

TABLE I. DESIGN PRINCIPLES AND OPERATIONALIZATION

Principle	Operationalization
External representations allow processing not possible/difficult to do in the mind.	The interface plots the graph of the equation/motion of the pendulum for various lengths and initial angles of the pendulum.
Cognition emerges from ongoing interaction with the world.	The interface is fully manipulable, ie, the learner can control the pendulum, equation and graph, to see how change in each affects the other elements.
The features of the world are used directly for cognitive operations, hence the interface should have all the features needed for integration of representations.	The interface has the physical system, equation and graph, along with different numerical values. The dynamicity of elements, and their interconnections, are made transparent, so that learners can integrate across spatial-numerical and dynamic-static modes.
The active self is critical for integration of features.	The exploration on the interface is guided by tasks which the learner must do.
Action patterns can activate concepts [33], hence actions and manipulations of the representations should be related to existing concepts.	The learner can interact with the pendulum by changing its length and initial angle by clicking and dragging the mouse. This interaction is meant to mimic the interaction with a real pendulum. The parameters in the equation -- moving up indicates increase in parameter, moving down indicates decrease. This is because it is known that numbers are grounded by associating small magnitudes with lower space and larger magnitudes with upper space [36].
The interface should allow coupling of internal and external representations.	The task requires student to match a given graph. Learners change the parameters of the pendulum/equation to generate the graph and visually match the task graph to their graph. This develops learner's imagination and coupling between their internal and the external representation.

##### A. Iteration 1: Action sequence for embodiment

The learner should perform the following action sequence in order for the integration of the representations to occur:

1. Manipulate the pendulum to a particular length and discharge the pendulum at an initial angle to begin the simulation. The equation of oscillation along with its graph is displayed simultaneously.
2. Change the period in the equation of oscillation. The pendulum's length changes and it oscillates at this new period. The graph is updated accordingly.
3. Manipulate the graph to change the frequency of the sinusoid. The change in frequency is translated to a change in length of the pendulum by the simulation which updates the equation and pendulum accordingly.
4. Repeat the same manipulations for initial angle.

Based on this sequence an initial interface was developed in Processing as shown in Fig. 1.

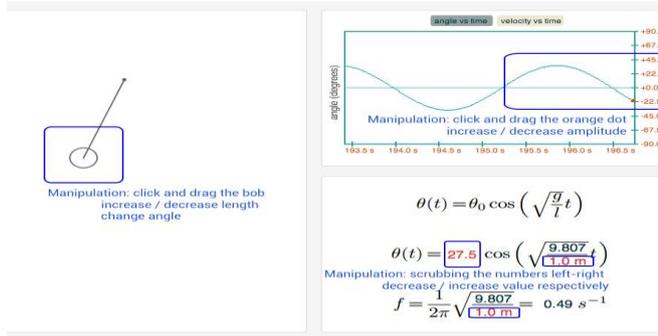


Figure 1. First iteration of interface design. All 3 components (pendulum, graph and equation) are manipulable.

### B. Iteration 2: Improving the interactions

In this iteration, we evaluated the initial interface against existing literature and theory, and modified the action sequence in order to:

1. Incorporate structured feedback to the learner during manipulation, as feedback is necessary to complete any action. So the pendulum appears first, the student can manipulate it and observe its behaviour using the play/pause button. Then the equation is added to the screen and the student can manipulate the equation and pendulum to see how they are connected. This helps the student understand the relation between the physical system and its equation. Finally the graph of the pendulum's motion is introduced.
2. To ensure too much is not happening at the same time, we removed the manipulation of changing the frequency and initial angle from the graph. The modified interface is shown in Fig. 2.

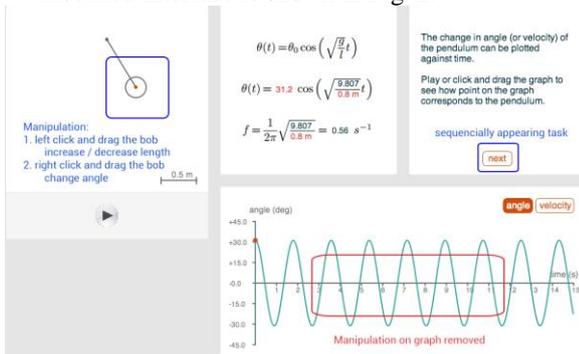


Figure 2. The second iteration of the interface only pendulum and equation manipulable.

### C. Iteration 3: Alignment with learning objectives

In this final iteration, we decided the learning objectives (LOs) of the interface, which guided the development of the tasks. In particular, our LOs for this interface were that the student should develop:

1. A dynamic understanding of equations.
2. An understanding of equations as controllers.
3. An integrated internal representation, consisting of the physical system, equation and graph.

In order to assess whether these LOs have been met, the students need to be able to do the following:

1. Mapping a physical system to a graph.

2. Mapping a physical system to an equation.
3. Mapping an equation to a graph.

We decided on a series of three tasks, requiring the student to manipulate the equation and pendulum to match a given graph. We hypothesized that these tasks were sufficiently complex, resulting in comprehensive exploration and manipulation of the interface by the student, so that the three representations are integrated.

Another significant change in this iteration was that the left and right “scrubbing” action to change the equation parameters was replaced by vertical sliders, as it is known that numbers are grounded by associating small magnitudes with lower space and larger magnitudes with upper space [36]. These interactions distinguish our interface from other variable manipulation simulations (eg. PhET [38]) in which the manner in which values are changed is not relevant, whether by slider, input box or multiple options. Our interface is specifically designed to make the learners do certain actions which mimic the behaviour of the system so that the system can be 'enacted' - the learning is through a form of participation with the system.

We then added the tasks to the interface and made the final interaction changes. After these, the interface was ready for piloting. A link to the final prototype used in the current study will be made available in the camera-ready version if this paper is accepted. A screenshot is shown in Fig. 3. The interface has six screens displayed one after the other as the participant clicks the next button. Screen 1 displays the manipulable pendulum (and instructions if applicable). Screen 2 shows the manipulable pendulum and equation/sliders (plus instructions if applicable), and screen 3 has the manipulable pendulum, equation/sliders and graph (plus instructions if applicable). Screens 4, 5 and 6 display pendulum, sliders/equation, graph and tasks 1, 2 and 3 respectively.

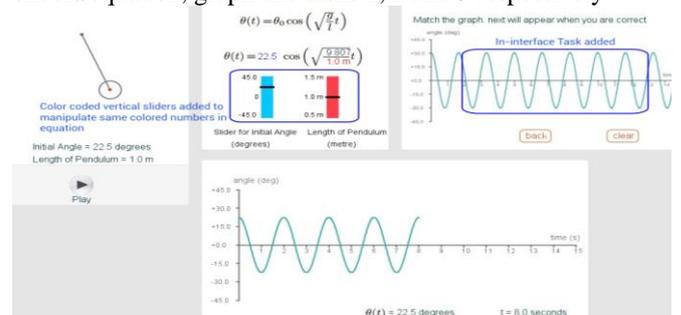


Figure 3. Final interface with sliders and On-interface task

## V. RESEARCH METHOD

We performed a usability and learnability study with the broad research goal of evaluating students' difficulties with the interface, particularly during exploration and performing tasks. Further, we wanted to understand what students do when they interact with the interface and whether they show behaviors, particularly manipulation/control behaviors, which can lead to integration of the equation/graph/model. As discussed in section II, our specific research questions were:

1. What is the difference in student exploration of the interface, particularly in terms of manipulation/control, in the text-guided and self-guided conditions?

2. What is the difference between student exploration of the interface before the tasks are presented and during tasks?

#### A. Sample

Our sample consisted of twelve (6 female) 7<sup>th</sup> grade school students from two urban schools in Mumbai. The sampling was based on convenience.

#### B. Procedure

We divided our sample into two groups, one which received an interface which had text instructions on it and the other which received an interface with no instructions. The latter group was only told that they could manipulate items on the screen with both left and right clicks of the mouse. The students were assigned to the two groups randomly. Students who indicated that they were not comfortable with computers were given a few minutes to practice with the mouse before the study began.

Once the student was shown the interface, he/she was allowed to work independently with it for as long as he/she wished, proceeding through the screens and tasks by clicking the “Next” button. The experimenter only intervened when students had a question and then only provided hints appropriate to their condition namely, text-guided or self-guided.

Once the student indicated that the tasks were completed or that they wished to quit, they were interviewed regarding their background, their impressions of the interface and administered an assessment task.

#### C. Data Sources

- Eye Tracker: Students’ eye movements were recorded using a Tobii X2-60 (static) eye-tracker, in order to capture how their loci of attention shifted as they explored the interface. This data could help decipher the dynamic process involved in integration.
- Video: Students were video recorded as they responded to the interview questions and worked on the assessment task.
- Researcher Observations: The researcher kept an unstructured log of student behaviours and facial expressions while they interacted with the interface, as well as their responses to the interview questions.
- Assessment task: The assessment task attempted to evaluate the extent to which students are able to imagine and mentally simulate the movement that they observed on the interface. The task consists of 6 questions, 3 multiple choice questions which ask students to imagine the position of the pendulum from the graph, and 3 marking questions which ask students to mark points on the graph corresponding to the pendulum’s position.

#### D. Data Analysis Technique

We analyzed the eye and mouse tracking data to unearth patterns of exploration in the text-guided and self-guided conditions, and before and after task presentation. Data of one student from the self-guided group was not included during analysis as she was disinterested and did not complete any of the tasks.



Figure 4. AOIs used for the analysis.

For analysis, separate segments of the interaction data of each participant for each screen were created. For a total of six screens (as described in Section IV.C), the segmentation yielded a total of six segments (or group of segments in case a participant switched between screens) per participant. Segments from all the participants for each screen were compiled to generate a scene that contained interaction data for all the participants for that particular screen. This yielded a total of six scenes (e.g. Scene for screen-1 had screen 1 interaction segments from all the participants, scene for screen-2 had all participants’ interaction data for screen 2, and so on). AOIs as depicted in Fig. 4 were then defined and generated across all the six scenes for statistical analysis. The following statistics were generated using Tobii studio per AOI, per screen, for each participant: (1) Total visit duration, (2) visit count, (3) mean fixation duration, (4) fixation count, and number of mouse-clicks. Statistics from individual participants for each screen were then tabulated into two groups depending on the nature of participants’ exploration (text-guided versus self-guided groups). Combined statistics for all the participants in that group were then used to plot graphs/figures/etc.

We also extracted student completion status, spent-time and accuracy data for interface tasks and accuracy data on the paper-pencil tasks.

Note that the statistical outputs do contain systematic errors to a certain degree arising out of calibration accuracy/precision differences among individuals, staggering of stimulus/task windows (for each individual as well as across participants), and loss of gaze data points due to blinks, proximity to the laptop screen, rapid head-movements, or moving out of the eye-tracking zone, etc. However, such errors are consistent across all the screens and participants.

## VI. RESULTS

We present a summary of student accuracy on the paper task and time taken by the students in each interface task in TABLE II.

As the sample size was small, and there were issues such as students giving up and some program errors in between, we decided not to focus on these end-point results. Instead, we analyzed the exploration process of the two groups, to get a qualitative understanding of the effect of guidance, as well as other aspects of the interface, on exploration and integration.

TABLE II. PAPER TASK ACCURACY AND AVERAGE TIME SPENT ON INTERFACE TASKS

Interface Task		1		2		3	
	Students	#	ave time spent (mins)	#	ave time spent	#	ave time spent
Guided	completed	5	8.934	3	19.58	3	11.51
	gave-up	1	30.42	1	5.34	2	8.94
	Prog error / no attempt	-	-	2	2.885	1	-
Self	completed	5	10.518	3	14.36	2	17.26
	gave-up	0	-	2	16.735	1	13.49
	Prog error / no attempt	-	-	-	-	2	-
Paper Task		Q1		Q2		Q3	
	Students	# Correct	# Wrong	# Correct	# Wrong	# Correct	# Wrong
	Guided	3	3	2	4	3	3
	Self	2	3	2	3	3	2

The time spent by a participant looking at each AOI in each screen, averaged over all participants in a group is presented in Fig. 5. One clear pattern is that the time spent looking at a screen goes up markedly during the task, compared to before it. Secondly, in pre-task exploration, the text guided group spends more time, on average, looking at the screen than the self-guided group. Though the look times of the two groups averaged across all tasks is comparable, the self-guided group looks more in task 1 and successively keeps looking lesser, while the self-guided group looks less in task 1 and successively keeps looking more. Further, this data shows that during the tasks in screen 4-6, both the groups spent a comparable duration on average, looking at the equation. Both groups also spent more time looking at the equation than the pendulum. However, the text-guided group spent more time looking at the pendulum than the self-guided group.

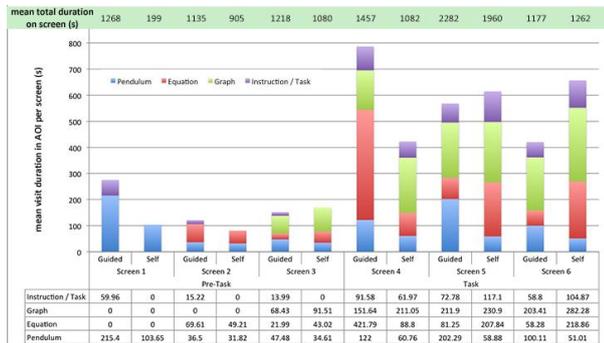


Figure 5. Average time spent by a participant in each Screen and the AOI of each screen in each of the two groups.

Next we present the number of mouse clicks in each AOI in each screen, averaged over all participants in a group in Fig. 6. This graph shows that there is a big increase in the number of mouse clicks once the task is displayed. Secondly, starting from screen 2 onwards, the self-guided group has more total number of clicks on each screen than the text-guided group. Further,

except for screen 4 in which both groups were comparable, the self-guided group has clicked more on the equation AOI (containing the sliders) than the text-guided group. The self-guided group has also clicked more on the graph AOI than the text-guided group.

The locations of the mouse clicks on the different AOIs of the interface pre-task and during the task are shown in Figs. 7 and 8. The first observation from Fig. 7 is that the text-guided group manipulates the pendulum more than the self-guided group, and the self-guided group manipulates the equation more than the text-guided group. Fig. 8 shows that the text-guided group uses both the pendulum and slider manipulation for all the tasks, however their usage of the pendulum was much lesser than the sliders. Further, they don't click much on the graph area. However we observe that the self-guided group doesn't use the pendulum for manipulation as much, reaching an extreme case in screen 6, in which they don't use the pendulum at all, instead performing the entire task with the sliders. Also we observe that this group clicks on the play button more than the other group, especially in screen 6. This group also clicks on the graph region several times in screen 4, but by screen 6 they stop this action.

These behavioural differences between the groups are summarized in Table III. We use these results together to answer our RQs. We look at RQ2 first, "What is the difference between student exploration of the interface before the tasks are presented and during tasks?" We find that in both groups the looking and the clicking increases after the task is presented and hence task-oriented exploration is better than "naked" exploration as it leads to more manipulation of the interface, which is one of our goals.

Now focusing on RQ1, "What is the difference in student exploration of the interface, particularly in terms of manipulation/control, in the text-guided and self-guided conditions?" As shown in Table III, based on the reported behavior of experts in the RC literature, we define good exploration as one in which there are more looks and clicks pre-task, and focused looking and few clicks during task. Comparing the exploration of the two groups against this canonical exploration, we find that the text-guided group has more looking and clicking in pre-task. During task however, neither group demonstrates this good exploration.

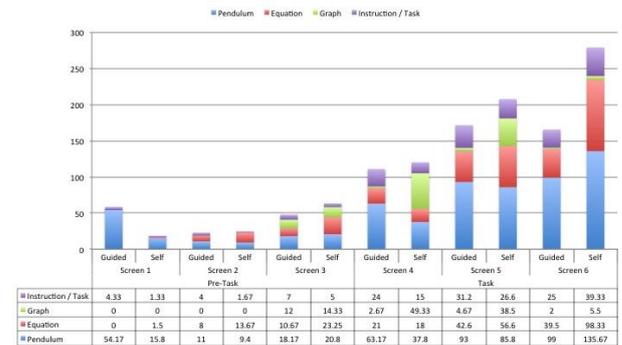


Figure 6. Mouse clicks on the interface in the two groups during-task.

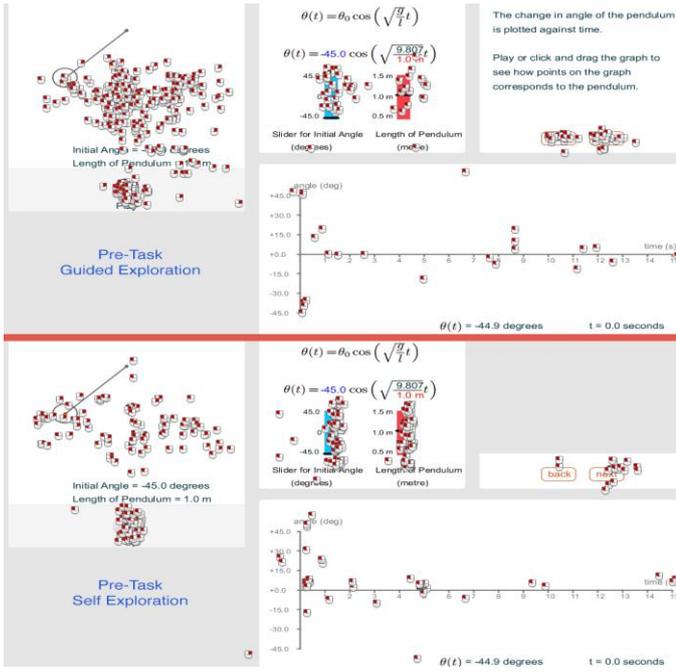


Figure 7. Average number of mouse clicks by a participant in each each screen of the interface in each of the two groups.

However since the text-guided group shows exploration closer to the good exploration in terms of less clicking, we could conclude that the text-guided group is better than the self-guided group. Overall, our tentative answer to RQ1, given the limited data from the pilot, is that text-instructions lead to better exploration than no instructions.

TABLE III. RELATIVE EXPLORATIONS OF PARTICIPANTS IN TEXT AND SELF GUIDED CONDITIONS

Exploration	Pre-task		During-task	
	Look	Click	Look	Click
Good			Focused	
Text-guided				
Self-guided				

## VII. DISCUSSION

The above analysis, results, interpretation, and decisions are presented to indicate our research approach to studying the RC problem, as well as making design decisions. These elements will change as we redesign the study based on this pilot and run more participants. Our preliminary analysis indicates that students in the text-guided condition looked at and manipulated all the elements of the interface (to different degrees) before and while working on the tasks. However the self-guided group focused on the equation (sliders) and graph, before and during the task. Thus the student exploration in the text-guided condition was more desirable for MER integration than in the self-guided condition. We also found that student exploration, both in terms of looking and clicking, increased after task presentation. Thus student exploration during task is more desirable than student exploration pre-task as student manipulation increases which we hypothesize facilitates integration.

The results indicate that students tend to focus on certain modes of representation (in this case the

equation) more than others and this has also been found in other studies with multi-representational displays [25]. However, our interface and study are the first targeting the development of RC by including interaction features emerging from embodied and distributed theories of cognition, and studying the development of RC using eye and mouse tracking.

The tracker data also revealed some limitations of our study design, particularly how the behavior of the unguided group was possibly skewed by introducing a familiar UI element (sliders) in the second screen. Particularly, we cannot be sure whether the students used the slider in the tasks because they understood the controller aspect of the equation or because they were familiar with this form of interaction. We need further studies after changing the sliders to an unfamiliar mode of interaction to evaluate the influence of a familiar UI element.

Our look and click data currently includes the looking and clicking on the play button, as well as the clear buttons, which need to be filtered out for a more fine-grained analysis. In further work, to better understand the thinking process of the participants, we will extract trajectory data, such as back and forth returns between two points/AOIs, their sequences, total distance traveled by the eye in each screen, overall structure of trajectories in the task phase, patterns of fixations on numerical versus spatial elements etc. We will also change the design so that the paper task is done on screen, so that eye movements during this phase can also be captured and analyzed, particularly to see if the movement patterns match the movement patterns during exploration, as found in previous studies [25]. Clearly, a lot more work needs to be done to get a better understanding of students' interactions with MERs and how MERs are integrated, the nature of RC and how RC could develop through interactive MERs.

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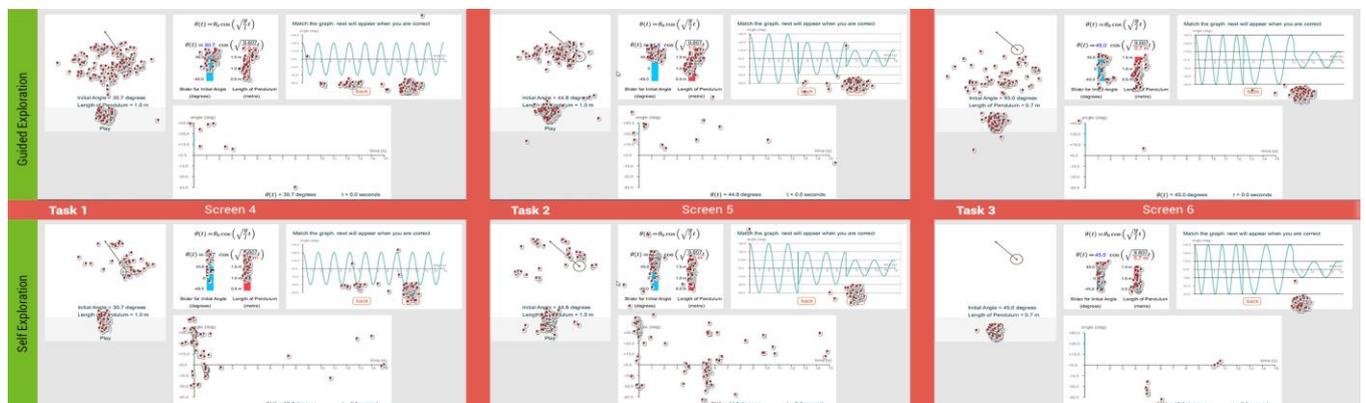


Figure 8. Mouse clicks on the interface in the two groups pre-task.